I shall discuss a positive solution to the following problem, obtained in a joint work with J. Boroński and J. Činč.

**Question (M. Barge, 1989 [8])** Does there exist, for every \( r \in [0, \infty] \), a pseudo-arc homeomorphism whose topological entropy is \( r \)?

Until now all known pseudo-arc homeomorphisms have had entropy 0 or \( \infty \). Recall that the pseudo-arc is a compact and connected space (continuum) first constructed by Knaster in 1922 [6]. It can be seen as a pathological fractal. According to the most recent characterization [5] it is topologically the only, other than the arc, continuum in the plane homeomorphic to each of its proper subcontinua. The pseudo-arc is homogeneous [2] and played a crucial role in the classification of homogeneous planar compacta [4]. Lewis showed that for any \( n \) the pseudo-arc admits a period \( n \) homeomorphism that extends to a rotation of the plane, and that any \( P \)-adic Cantor group action acts effectively on the pseudo-arc [7] (see also [10]). We adapt Lewis’ inverse limit constructions, by combining them with a Denjoy-Rees scheme [1] (see also [9], [3]). The positive entropy homeomorphisms that we obtain are periodic point free, except for a unique fixed point.

I am going to present various results related to the problem, to conclude with a discussion of its solution.

**References**


